Splash 2957: Introduction to Combinatorial Game Theory **Spring 2013**

Preliminary Notes

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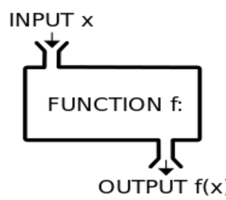
1.1 Function

Formally, a function is a relation that maps the input set into the output set, with the property that the same input will result in the same output all the time (ie the output is not random). Informally, you can treat a function as a black-box. You can ask it certain questions, and the black-box will return some answer after some internal calculation based on your answer. Asking the machine the same question will always result in the same answer.



As a concrete example, consider the quadratic curve $y = x^2$. We can think of this as a function $f(x) = x^2$, where given a certain value x, we get the square of that value.

Note that a function can return anything as long as you define it properly. For example, you can define $f(x)$ to return a piece of cake if x is even, and a lie otherwise.

1.2 Binary Operations

A binary operator can be thought of as a function. It takes in two values and returns a single value. The 4 basic operations of addition, subtraction, multiplication and division are examples of binary operators. For example, the addition operator + takes two values a and b as input, and returns their sum as output.

You can define your own operators. For example, define $x\Delta y = x^2+y^2-xy$. Then $3\Delta 5 = 3^2+5^2-3\times 5 = 24$.

1.3 Change of Base

"There are 10 kinds of people; those who understand binary and those who don't."

The number system we are most familiar with is the decimal (base 10) number system. Each number is expressed in terms of powers of 10. For example, the number 2187 is expressed as $2 \times 10^3 + 1 \times 10^2 + 8 \times 10^1 + 7 \times 10^0$. However, we can express 2187 in other number systems as well. In binary (base 2), 2187 is equal to 100010001011_2 (the subscript shows what base it is), or $2^{11} + 2^7 + 2^3 + 2^1 + 2^0$. The i^{th} digit counting from the right denotes 2^{i-1}. In ternary, 2187 is 10000000_3, as $2187 = 3^7$.

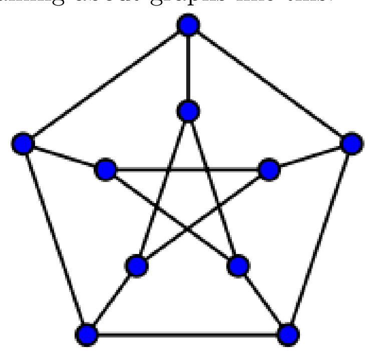
We will mostly be dealing with binary bases, so let's see how we can convert decimal numbers into binary numbers. Given a number, we continously remove the highest power of 2 we can from the number until we get 0. For example, the greatest power of 2 smaller than or equal to 2187 is 2048, or 2^{11}. Subtracting that from 2048, we get 139. The greatest power of 2 smaller than or equal to 139 is 128, or 2^7. We continue with this process to get $2^3, 2^1$ and 2^0.

Try and see if you can convert the following numbers into binary.

a) 1729 b) 281 c) 849 d) 8192

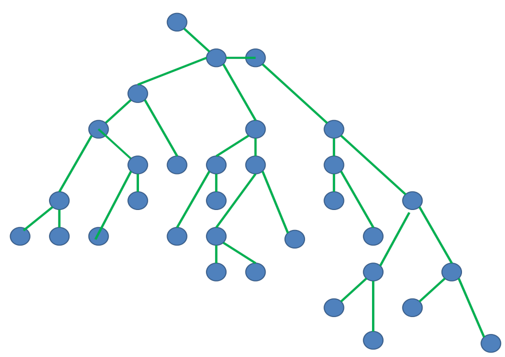
1.4 Graphs and Trees

We will learn some concepts of graph theory here. We do not need to any knowledge of graphs, but it's good to know what graphs are and how they function. To clarify, we are not talking about line graphs that we plot on the cartesian plane. We are talking about graphs like this:



Graphs are mathematical objects represented by vertices and edges. Vertices are the little dots that appear at the intersections. Edges are the lines that join up the vertices. It is a very simple construct really, but there are many nice properties about graphs that we unfortunately will not explore. The above graph is a special graph called a Petersen graph. Notice that each vertex in the graph is connected to exactly 3 edges.

During our session, we will mostly encounter trees. We can guess why it is called a tree by looking at the image below:



If we flip the image along the horizontal axis, then the edges of this graph look like the branches of a tree. Trees have a very special property; there are no cycles. That is, you cannot travel through the edges and return back to your starting vertex.

That's all we need to know for now. See you on Sunday!